Inductance Extraction of Multilayer Finite-Thickness Superconductor Circuits

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***Abstract*—In this paper, an efficient numerical technique is presented for inductance extraction and current calculation in multilayer planar superconductor microelectronic circuits. The rigorous definition of a three-dimensional problem based on London equations and stream function is presented. The finite thickness of conductors is taken into account. The results can be directly applied to perfect and extended to normal conductors.**

***Index Terms*—Finite-element method, impedance, inductance, normal conductors, perfect conductors, sheet current, stream function, superconductivity.**

I. Introduction

In this paper, the problem of three-dimensional (3-D) electromagnetic modeling of superconductive multilayer planar multiconnected microelectronic circuits is considered. These circuits can be digital, superconducting quantum interference devices (SQUIDs), some modern high-T_c [1], [2], or microwave devices.

The shape of modern circuits can be very complex. Magnetic field in such layouts essentially has a 3-D structure. This circumstance practically excludes the implementation of simplified two-dimensional (2-D) models, transmission line or planar [2].

One of the problems we meet is that conductors cannot be accepted as infinitely thin because the thickness of conductors and dielectric layers and London penetration depth are of same order of magnitude [3], [9].

Earlier, the problem of 3-D inductance extraction for superconductors was considered in [3]–[6]. These works are based on a technique that is known for normal conductors [7] as the partial-element equivalent-circuit (PEEC) method. For superconductors, the most complete realization of this method is presented in [3]. It was found [3], [2] that practical calculations can be very time and memory consuming even with fast solvers [8]. Thus, it was necessary to continue the development of a more efficient numerical technique.

The implementation of stream function (or T-function or vector potential representation) can overcome these problems.

For inductance calculation of perfectly conducting foils (sheet currents), a stream function was used in [10]. In our case, methods [10] are inadequate because we consider more complicated structures. Moreover, the numerical technique [10] is not accurate enough and can be improved.

In this paper, we propose a new numerical approach, which can improve the PEEC method for planar objects. The rigorous definition of the problem based on London equations and stream function is presented. All excitation currents are defined by simple boundary conditions of the first kind in the way similar to Laplace equation. The matrix of self and mutual inductances is defined using full energy.

The numerical technique of current research is a further development of [11]. The finite-element method on a triangular mesh with linear finite elements is used. For these finite elements, the current density is simulated by circulation currents with piecewise-constant density. This numerical technique is proved enough and leads to the system of linear equations with positively definite symmetric dense matrix. The results of calculations for a substantially 3-D problem, strip over hole in ground plane, are presented.

The numerical technique and program of current research are not restricted to only superconductor circuits. Setting London penetration depth to zero, the program can be implemented for quasi-static analysis of perfectly conducting foils with possible application to various transmission-line discontinuities, baluns, and inductors.

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II. Preliminaries

In this paper, we study the currents in conducting layers separated by layers of dielectric. Let t_m be the thickness of conducting layers and d_k be the thickness of dielectric layers, and k, m be the numbers of the layers. Conducting layers can contain few single-connected conductors of arbitrary shape. Let the number of conductors in all layers be N_c and the total number of holes in all conductors will be N_h. Each conductor can have current terminals.

For a large class of microwave and digital circuits, it can be assumed [2], [3]

$$d_k \ll l \quad t_m \ll l \quad \lambda_m \sim t_m \tag{1}$$

where l is the typical lateral size of the circuit in plane $(x,\, y)$, and λ_m is the London penetration depth.

Each conductor occupies space domain $V_m = S_m \times [h_m^0,\, h_m^1]$, $m = 1, \ldots, N_c$. The 2–D domain S_m is the projection of the conductor on the plane $(x,\, y)$. We call the boundary of the conductor ∂S_m the boundary of the projection S_m. Let $\partial S_{h,\,k}$ be the boundary of the hole with number k, and $\partial S_{\text{ext},\,m}$ be the external boundary of the mth conductor. We assume that all current terminals are on the external boundary of the conductors.

The magnetic field is excited by currents circulating around holes and currents through chains of terminals on the conductors. Let N_t be the number of these terminal chains. Thus, the total number $N = N_t + N_h$ of excitation currents is the dimension of the inductance matrix. As the extraction of the inductance matrix includes the current simulations, we will concentrate on the problem of inductance calculation.

For further convenience, let P, P_0 stands for points in 3-D space, and r, r_0 stands for points on plane. Also, consider differential operators $\partial_x = \partial/\partial x$, $\partial_y = \partial/\partial y$, and $\nabla_{xy} = (\partial_x,\, \partial_y)$.

III. London Equations for Conductors of Finite Thickness

The basic equations for further consideration are static London equations [1]. Let $\vec{j}$ be the current density, $\vec{B}$ be its magnetic field, and λ be the London penetration depth. The basic equations are

$$\mu_0 \lambda^2 \nabla \times \vec{j} + \vec{B} = 0, \tag{2}$$

$$\nabla \times \vec{B} = \mu_0 \vec{j}. \tag{3}$$

From (1) and x, y components of vector equation (2) follows $j_z \approx 0$, $\vec{j} \approx \vec{j}(x,\, y)$. Then z-component of (2)

$$\mu_0 \lambda^2 \Big(\partial_x j_y(P_0) - \partial_y j_x(P_0) \Big) + B_z(P_0) = 0 \tag{4}$$

is the governing equation for the current in the plane $(x,\, y)$.

Consider the sheet current density $\vec{J}_m(r)$

$$\vec{J}_m(r) = \int_{h_m^0}^{h_m^1} \vec{j}(P)\, dz, \qquad r \in S_m. \tag{5}$$

The magnetic field in (4) is calculated by means of average current density $\vec{J}_n(r)/t_n$ and the Biot–Savart formula

$$\vec{B}(P_0) = \frac{\mu_0}{4\pi} \sum_{n=1}^{N_c} \int_{V_n} \frac{1}{t_n} \vec{J}_n(r) \times \nabla_P \frac{1}{|P - P_0|} \, dv_P. \tag{6}$$

Consider London penetration depth for films

$$\lambda_m^s = \lambda_m^2 / t_m. \tag{7}$$

Averaging (4) over the thickness of conductors, we obtain the following equations for the sheet currents in conductors

$$\lambda_m^s (\partial_x J_{m,y}(r_0) - \partial_y J_{m,x}(r_0)) \tag{8}$$

$$+ \frac{1}{4\pi} \sum_{n=1}^{N_c} \iint_{S_n} \left(\vec{J}_n(r) \times \nabla_{xy} G_{mn}(r, r_0)\right)_z ds_r = 0 \tag{9}$$

where $r_0 \in S_m$, $m = 1, \ldots, N_c$, and

$$G_{mn}(r, r_0) = \frac{1}{t_m t_n} \int_{h_m^0}^{h_m^1} dz_0 \int_{h_n^0}^{h_n^1} \frac{1}{|P - P_0|} \, dz. \tag{10}$$

Equation (9) must be completed by the charge conservation low $\nabla \cdot \vec{J}_m = 0$, $m = 1, \ldots, N_c$. Kernels $G_{mn}(r, r_0)$ (10) can be calculated analytically and have logarithmic singularity if $r = r_0$. For small λ, calculations with this singularity can be unstable. Therefore, we substitute both of the one-dimensional integrals in (10) by quadrature formulas of rectangles or trapezoids.

For $h_n \in [h_n^0, h_n^1]$, the formula of rectangles gives us the following kernels:

$$G_{mn}(r, r_0) = 1 \Big/ \sqrt{|r - r_0|^2 + (h_m - h_n)^2}\,. \tag{11}$$

Obviously (11) is an infinitely thin current sheet approximation where sheets have heights h_n. In this case, values h_n are fitting parameters of the method.

To avoid fitting parameters that have a strong influence on the accuracy of the method, the trapezoid formula is used. For $m = n$, the kernel has the form

$$G_{mm}(r, r_0) = \frac{1}{2} \left(\frac{1}{|r - r_0|} + \frac{1}{\sqrt{|r - r_0|^2 + t_m^2}} \right). \tag{12}$$

This approach show good precision and numerical stability for problems with finite thickness of conducting and dielectric layers.

IV. Stream Function

For the sheet current, well-known stream function representation is used. Stream function (T-function) $\psi_m(r)$ is defined for each single-connected conductor on the base of charge conservation law. In our case,

$$J_{m,x}(r) = \partial_y \psi_m(r) \quad J_{m,y}(r) = -\partial_x \psi_m(r). \tag{13}$$

Or, if $\vec{\Psi} = (0, 0, \psi(r))$, then $\vec{J}_m = \nabla \times \vec{\Psi}$.

$\psi_m(r)$ has the sense and dimension of a full current. Let $\Gamma \in S$ be any open curve in S_m with the origin r_0 and end r_1. The full current through this curve is then $I(r_0, r_1) = \psi_m(r_1) - \psi_m(r_0)$. The full current does not depend on a specific curve joining P_0 and P_1.

Let us introduce the necessary agreements concerning functions $\psi_m(r)$. We assume the normal current distribution through terminals is homogeneous and $\psi_m(x, y) = 0$ on a nonterminal part ∂S_m^0 of boundary S_m. We define the total current circulating around a hole in S_m as the total current through any curve joining ∂S_m^0 and the boundary of the hole. The total current does not depend on the choice of the joining curve.

Substituting (13) into (9), we obtain the set of equations for functions $\psi_m(r)$, $m = 1, \ldots, N_c$

$$-\lambda_m^s \Delta \psi_m(r_0) + \frac{1}{4\pi} \sum_{n=1}^{N_c} \iint_{S_n} \left(\nabla \psi_n(r), \nabla_{xy} G_{mn}(r, r_0)\right) ds_r = 0. \tag{14}$$

Let $I_{h,k}$ be the full currents circulating around the holes $k = 1, \ldots, N_h$, the boundary conditions for (14) are then

$$\psi_m(r) = I_{h,k}, \qquad r \in \partial S_{h,k};\ k = 1, \ldots, N_k \tag{15}$$

$$\psi_m(r) = F_m(r), \qquad r \in \partial S_{\text{ext},m};\ m = 1, \ldots, N_c. \tag{16}$$

Function $F_m(r)$ is defined by the properties of $\psi(r)$ and terminal current distribution. $F_m(r)$ are linear in the limits of terminals, are constant on the nonterminal boundaries, and $F_m(r) = 0$ on ∂S_m^0, $m = 1, \ldots, N_c$.

Equation (14), together with boundary conditions (15) and (16), completely define the current distribution in the circuit and allow us to define the inductance matrix.

Perfect conducting foils are incorporated in (14)–(16) if $\lambda_m^s = 0$ and kernels are as shown in (11).

V. Matrix of Inductances

For the definition and calculation of an inductance matrix, we use an energy approach. The functional of full energy has the form [1]

$$E = \frac{1}{2} \sum_{n=1}^{N_c} \int_{V_n} \left(\mu_0 \lambda_n^2 J(P)^2 + \vec{J}(P) \cdot \vec{A}(P)\right) dv \tag{17}$$

where $\vec{A}$ is a vector potential, $\vec{B} = \nabla \times \vec{A}$. For the stream function, the approximate expression for full energy is

$$E = \frac{\mu_0}{2} \sum_{n=1}^{N_c} \iint_{S_n} \lambda_s^n (\nabla \psi_n)^2 \, ds_n + \frac{\mu_0}{8\pi} \sum_{n=1}^{N_c} \sum_{m=1}^{N_c} \iint_{S_n} ds_n \iint_{S_m} (\nabla \psi_n, \nabla \psi_m) G_{mn} \, ds_m. \tag{18}$$

Let $\vec{I} = (I_{h,1}, \ldots, I_{h,N_h}, I_{t,1}, \ldots, I_{t,N_t})$, where $I_{t,i}$ are full currents through terminal sequences $i = 1, \ldots, N_t$ ($N = N_h + N_t$). As the problem (14)–(16) is linear, then (18) is a positive quadratic form with respect to $\vec{I}$. It means that there is an $N \times N$ symmetric positive–definite matrix L

$$2E = (L\vec{I}, \vec{I}). \tag{19}$$

Matrix L is the matrix of self and mutual inductances.

Inductance matrix L allows us to calculate fluxoids related to holes and terminal currents. Fluxoid Φ is the contour integral over a closed or open [11] curve $\Gamma \in S_m$

$$\Phi = \int_\Gamma \left(\mu_0 \lambda_s^m \vec{J}_m + \vec{A}\right) \cdot d\vec{l}_\Gamma. \tag{20}$$

Fluxoids $\vec{\Phi} = (\Phi_1, \ldots, \Phi_N)$ can be easily calculated as $\vec{\Phi} = L\vec{I}$ [11]. In contrast with [10], it is not necessary to introduce special contours for calculation of these values.

VI. Numerical Technique

The method of computation of elements of the inductivity matrix consists of the solution of sequence of boundary value problems (14)–(16) with special excitation currents and calculation of the free energy.

As $\vec{\Phi} = LI$, it is possible to find L calculating fluxoids (magnetic fluxes for perfect conductors [10]). In our case, it appeared to be more complicated, more time consuming, and less numerically stable than a full-energy approach (18), (19).

The finite-element method for (14)–(16) is the extension of the method [11]. For simplicity, we consider the case of one conductor. The extension on a multiconductor case is straightforward. Below, the conductor indexes are omitted.

The bilinear form $a(u, v)$ for "weak" [12] formulation of the problem (14) is

$$a(u, v) = \lambda^s \iint_S (\nabla u(r), \nabla v(r))\, ds + \frac{1}{4\pi} \iint_S ds_0 \iint_S (\nabla u(r), \nabla v(r_0)) G(r, r_0)\, ds. \quad (21)$$

The principal value integral in (14) was integrated by parts. Form $a(u, v)$ is symmetric and for thin enough conductors positively definite because for $u = v$ it coincides with the expression for the full energy (18).

For the triangulation of S, let I be the set of indexes of internal points of the mesh and J be the indexes of all nodes including boundary nodes. The unknown function $\psi(x, y)$ is approximated by linear finite elements [12]. The prescribed boundary values of $\psi(x, y)$ are taken into account by the following approximation:

$$\psi(x, y) \approx \psi^h(x, y) = \sum_{j \in J} \psi_j^h u_j^h(x, y) \quad (22)$$

where ψ_j^h are the approximate values of $\psi(x, y)$ in the nodes of mesh and $u_j^h(x, y)$ are the basic functions of finite-element interpolation [12]. Setting in (21) $u(x, y) = \psi^h(x, y)$, $v(x, y) = u_i^h(x, y)$ one can derive the following system of linear equations:

$$\sum_{j \in J} a(u_i^h, u_j^h) \cdot \psi_j^h = 0, \qquad i \in I. \quad (23)$$

Equation (23) can then be rewritten as system of linear equations for ψ_i^h, $i \in I$ with a symmetric dense matrix where the nonzero right part is formed by terms with prescribed boundary values $\psi(x, y)$ (15), (16). We solve finite-element equations using a Cholesky CLAPACK routine.

The program implementation of finite-element method meets some difficulties. Expression (21) contains quadruple integrals. Calculation of these integrals over triangles is the most CPU time-consuming part of the algorithm. Singular integrals are calculated analytically [13]. Nonsingular integrals are calculated numerically. The order of quadratures depends of the proximity of mesh cells.

The program includes an internal mesh generation preprocessor. There are no restrictions on the shape of the circuits.

Our program is written on C. Except visualization, the program is platform independent. It is possible to convert AutoCAD draw to input file format of our program.

VII. Examples

A. Strips Over Ground Plane

Recently, the survey of inductance extraction programs was published [2]. The preliminary version of our program ML participated in this comparison. In completing of [2], consider a simple problem: strip over ground plane. The strip is 10-μm width, 40-μm length, and $d_0 = t_1 = 0.15$ μm. The ground plane is 0.1-μm thick and $\lambda = 0.09$ μm. The exact inductance is 0.773/10 μm. Our result is 0.77/10 μm; the error is 0.1%. These results were obtained on the mesh with 693 unknowns with cells diameter of about 1 μm. Total CPU time (WINDOWS NT, P-166) was 200 s. The ground plane was meshed and magnetic field penetration was taken into account.

Our program appeared to be fast and accurate enough to calculate ground-plane effects. Treating the ground plane as a "mirror plane" leads to a difference in results up to 20% [2].

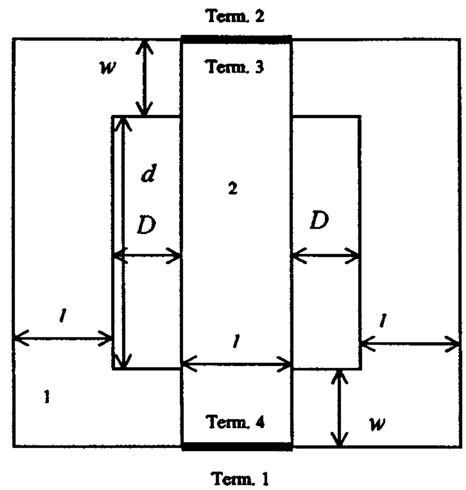

Fig. 1. Strip over hole in ground plane (top view).

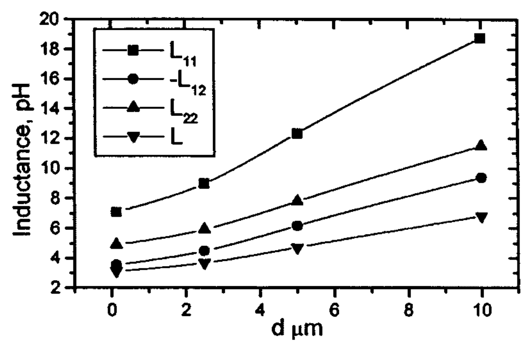

Fig. 2. Inductance coefficients for a strip line over a hole in the ground plane obtained by the finite-thickness approach.

B. Strip Over a Hole in the Ground Plane

Consider essentially a 3-D two-layer problem, strip over hole in ground plane (Fig. 1). This structure is a typical element of many circuits. The hole leads to an increase of the inductance of the strip.

The inductance matrix has dimension two. L_{11} is the self inductance of hole, L_{22} is the self inductance of the current path, which includes a strip and ground plane with a current flowing from one side of the hole, and L_{12} is mutual inductance. The inductivity of the strip L is calculated under the condition of zero fluxoid in the hole $L = L_{22} - L_{12}^2/L_{11}$.

For a stretched hole ($d \ll l$ in Fig. 1), L can be approximately calculated by means of 2-D transmission-line programs [9]. In [9], the results of measurements and calculations for the next values of parameters are presented: $l = 12.7$ μm, $D = 5.1$ μm, $t_0 = 0.32$ μm, $d_0 = 0.41$, $t_1 = 0.223$ μm, and $\lambda = 0.125$ μm. Setting $d = 50$ μm, $w = l$ (Fig. 1) for the finite-thickness conductors, we obtain $L = 0.336$ pH/μm. From [9], $L \approx 0.34$ pH/μm. The results coincide with good accuracy.

For a square or narrow gap ($d \ll 2D + l$), two-dimensional approximation is irrelevant. Our results for finite-thickness conductors are shown in Fig. 2, where $t_0 = 0.1\ \mu$m, $d_0 = 0.15\ \mu$m, $t_1 = 0.2\ \mu$m, $\lambda = 0.09\ \mu$m, and $l = 5$, $D = l/2$, $2w + d = 4l$. The values of L_{22} and L do not converge to the inductance of the strip without a hole because the current in ground plane needs to flow around the cut.

VIII. Conclusions

In this paper, we have proposed a new numerical technique for analyzing planar multilayer superconductor circuits. The developed program allows us to calculate inductances for realistic 3-D circuits yielding very reasonable CPU time.

Our program can be applied for calculation of inductances of perfect conductors simply setting to zero the London penetration depth.

The results of this paper can be extended to the case of impedance calculation of normal conductors.

The program can be implemented as a component in electromagnetic computer-aided design (CAD) complexes.

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A Design of the Ceramic Chip Balun Using the Multilayer Configuration

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***Abstract*—This paper presents the design method and performance characteristics of a chip-type balun using a multilayer structure. The design method for a chip-type balun is based on the lumped-element equivalent circuit of quarter-wave transformer. The proposed design method and equivalent circuit can make it easy to design the ceramic multilayer chip-type balun. The size 2012 and 3216 chip-type baluns were designed and fabricated using the proposed design method and the equivalent-circuit model of a quarter-wave transformer. Fabrications and measurements of designed chip-type baluns show smaller size than conventional chip-type baluns and good agreement with simulated results.**

***Index Terms*—Multilayer structure, quarter-wave transformer, 2012 and 3216 chip-type balun.**

I. Introduction

Multilayer configurations can provide several advantages in the integration and compaction of RF and microwave components, circuits, and systems. Another reason for employing the multilayer configurations is that several circuits function such as baluns, coupler, etc., which are difficult to realize in a single-layer planar configuration, can be obtained conveniently in two- or multiple-layer configurations. Several kinds of multilayer passive components, such as filters, couplers, and balun, have been developed and each design methods and fabrication procedures have been reported. The baluns and couplers require no suspended substrate techniques. Hence, they can be easily incorporated in the design of a variety of components such as mixers, multipliers, and push–pull class-B amplifiers [1]–[4].

In this paper, an approach for the design of a chip-type balun using a multilayer structure is presented. The presented design method for the chip-type balun is developed using the equivalent circuit of a quarter-wavelength transformer. By employing the proposed design method and equivalent-circuit model, the designed multilayer chip balun configuration can be made more compact and flexible, providing better performances. The designed chip-type balun was realized by implementing the multilayer chip inductors and capacitors. Each lumped element was implemented using the ceramic material and Ag metal layers. The dielectric constant was chosen to be six for the implementation.

This paper presents the experimental results showing the changing of the frequency characteristics with a lumped-element value. Simulation results and experimental measurements for the designed chip-type balun show a validation of the proposed design method.

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